## LETTERS TO THF EDITOR

REPLY TO THE LETTER OF PROFESSOR A.A. GUKHMAN
B. A. Shulyak

A "Letter to the Editor" of Professor A. A. Gukhman in response to critical remarks concerning the views on a number of problems of similarity theory expressed in my book Physics of Waves at the Surface of a Free-Flowing Medium and a Liquid (Nauka, Moscow, 1971) was published in the October issue of this journal.

In Professor A. A. Gukhman's letter there are six points of objection in which he indicates the inaccuracy of critical remarks addressed to his book Introduction to Similarity Theory of 1963 (abbreviated IST). A detailed review of his book (IST), his earlier work Determination of the Number of Similarity Criteria (Vol. 6, Zap. Kazakh. Gosuniversiteta, 1941), M. V. Kirpichev's book Similarity Theory of 1953, and other primary sources was given in my reply to Professor A. A. Gukhman of November 27, 1972.

In this connection I ask the Editor to publish my letter with the reply in brief form.
In point 1 of the objections A. A. Gukhman pointed out that in his book the $\pi$-theorem does not pertain to similarity theory.

In my reply it was indicated that not only the $\pi$-theorem but also the entire method of dimensional analysis in IST pertains to similarity theory with all the resulting consequences. In fact, on p. 4 of IST we read: "The book is intended (and this is reflected in its name) as an introduction to similarity theory ..." and "Similarity theory and dimensionality analysis are considered as a single unit..." On p. 250: "... the basis of the general method of the determination of the number of criteria ( $\pi$-theorem) iliuminates the fundamental bases of a unified analysis." On p. 47: In this connection it is accurate to call the $\pi$-complexes similarity criteria ..." Finally, on p. 54: "We note once again that some of the $\pi$-complexes (namely those which have the greatest importance for theoretical studies and calculations) have begun to be denoted by the first letters... of the names of the scientists ... such as Re for the Reynolds number."

In point 2 of the objections it is indicated that in IST the similarity criteria are not derived from outside sources, apart from equations of motion.

I showed that in IST the criteria are obtained also from boundary conditions, which are not equations of motion (see the terminology and definitions in my book, Part 11). For example, on p. 21 of IST we read: "The indeterminacy arising in the study of an equation not connected with the supplementary conditions is caused by deep physical reasons. In it the re appears the fact that the process is insufficiently fully determined by the fundamental equations of the problem." Thus, knowledge, to which one must add the fundamental equations, represents the conditions of uniqueness of the solution. However, the question of how these conditions should be constructed and just what information they should express remains open."

In point 3 of the objections it is indicated that there is no criterial equation in IST.
In my reply to is shown that on pp. $59,103,119,133,162,171$, and others the equations presented are criterial, although they are called generalized equations. In fact, concerning the procedure of formulating the arguments of the se equations it is said quite definitely on pp. 247-248: "Thus, using the apparatus of dimensional analysis the problem of the structure of the generalized variables is solved through the following scheme ..."

It is indicated in points 4 and 5 that in IST the Reynolds number is not allotted the role of a universal criterion and that different criteria are not supplied to the same processes.

I showed that on p. 127 and the following pages the "universality" of the Reynolds number follows from an analysis of the problem of the self-similar motion of a liquid at low relative values of the viscosity. (An analysis of the solution of the equations of a heavy liquid is presented in my reply to Professor A. A. Gukhman of November 27, 1972 and the criteria not taken into account in IST are indicated. - B. Sh.)

This is also seen from the identification of the $\pi$-factors with the similarity criteria (see the replies to point 1 and others). For this very reason the similarity criteria also prove to be "applicable" to pro-
cesses to which they have no relation. For the same reason different criteria are "correct" for the same processes. Actually, since according to the $\pi$-theorem the choice of the fundamental units determines its set of dimensionless $\pi$-factors (and according to IST these are the similarity criteria), the use of $\pi$-factors for this purpose leads to an arbitrary description of the problem - by any choice of $\pi$-factors.

In point 6 an objection is expressed against the criticism of the inverse similarity theorem presented in a footnote to p. 50 of my book. In connection with this Professor A. A. Gukhman writes: "Of course, there is nothing of the kind in my last book."

In fact, the inverse similarity theorem is not mentioned in Professor A. A. Gukhman's 1963 book. However, in the footnote on p. 50 of my book indicated by Professor A. A. Gukhman there is no reference to A. A. Gukhman's book Introduction to Similarity Theory of 1963. My book is talking about M. V. Kirpichev's similarity Theory, published in 1953 , in which the inverse similarity theory under discussion is assigned No. 3 (the $\pi$-theorem is assigned the number two!). But M. V. Kirpichev calls Theorem No. 3 the Kirpichev-Gukhman theorem and says that he formulated it jointly with A. A. Gukhman. Therefore the indicated reference in my book to this theorem with citation of Professor A. A. Gukhman's name cannot raise objections.

In conclusion, I must note that in my opinion at least some of A. A. Gukhman's objections are evidently caused by simple misunderstanding, which is most clearly seen from point 6 of his objections. Of course, a difference of opinions, conviction, or even habituation to established concepts may appear here in part. The latter is probably the main reason, although I do not undertake to assert this.

